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Zero - Hopf Bifurcation in a 3D Jerk System

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***Abstract*— The paper provides an introductions to bifurcation of three dimensional quadratic Jerk system and Zero - Hopf Bifurcation and their local behaviour in Jerk dynamical systems.**

***Keywords:* Jerk systems, zero-hopf bifurcation, chaotic systems, stability.**

I. INTRODUCTION

The study of peroidic orbital of differential system are one of the main objective of the qualitative theory of the differential equations.

A way to finding periodic orbits is through the Hopf Bifurcation which in R^3

takes place when a periodic orbit bifurcate from an equilibrium point whose linear part has eigen values $\lambda \neq 0$

And $\pm\omega\,\iota$

Where $\iota = i$ & $\omega > 0$

And moving the parameters of the differential systems this equillibrium changes its kinds of stability .

Equilibrium play an important role in studying the dynamics of non-linear differential systems for example one method of determining Chaos in the thrilnikov's method which is based on the homoclinic orbit of a saddle focus ensurring existing of a horsethere in the neighbourhood of this orbit and consequently the birth of chaotic motion.

There are many examples existingibly have rich dynamic behaviour see for instance the lorentz's Systems

$$\dot{x} = y$$

$$\dot{y} = z$$

$$\dot{z} = a_2 y + a_3 z + a_4 x^2 + a_5 xy + a_6 xz + a_7 yz$$

$$where\ a_3^2 + 4a_2 < 0, a_4 \neq 0$$

For which the resulting periodic solution is unstable.

We study the Zero- Hopf bifurcation at origin of the following systems

$$\dot{x} = y$$

$$\dot{y} = z$$

$$\dot{z} = (a_3 + a_4 x)z$$

$$+(a_2 + a_5 x)y - x^2$$

provided an estimate of the periodic solutions and their kind of stability.

Braun and mereu obtained a Zero -Hopf Bifurcation in a chaotic Jerk system.Despite their simple form Jerk system provides examples of chaotic behaviour [1], [2], [3], [5]. Bifurcation in the dynamics of Jerk systems are also analysed [1], [2], [3], [4], [5].

Besides the chaotic behaviour another investigation topic is the local bifurcation of Jerk systems particularly the Zero -Hopf Bifurcation and the number of limit cycles which are born from this bifurcation[Braun and mereu,2021], [Guo et al, 2023,sun et al 2022].

In [messias &Silva,2020] Condition on the parameter in order to guarantee the non-chaotic behaviour for some classes of Jerk systems are obtained.

A zero -Hopf equillibrium p of a differential system in R^3

Is an equilibrium point with Eigen values $\lambda \neq 0$

and $\pm\omega i$

with $\omega > 0$

A Zero- Hopf bifurcation takes place when one or several periodic orbits bifurcate from the equillibrium p

Hence the parameter of the system move.

Consider a System defined by

$$\dddot{x} + h(\dot{x}, x, \mu) = 0$$

Where h

Smooth and μ

is a parameter.

After a linear transformation of parameters we can assume that as μ increases from below zero to above zero the origin turns from a spiral sink to to a spiral source.

Now for $\mu > 0$

We perform a Perturbation Expansion using two timings.

$$x(t) = \epsilon\, x_1(t, T) + \epsilon^2 x_2(t, T) + \epsilon^3 x_3(t, T) + \cdots \ldots \ldots \ldots$$

Where

$$T = vt$$

is slow time(thus two timings) and ϵ and v are functions of μ

By an argument with harmonic balance we can use $\epsilon = \mu^{\frac{1}{2}}, v = \mu$

Hence plugging in $x(t_1)$

to

$$\dddot{x} + h(\dot{x}, x, \mu) = 0$$

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And expanding upto the ϵ^3

Order we would obtain three ordinary differential equations in x_1,x_2,x_3.

The first equation would be of the form

$$\partial_t\ x_1 + w_0^2 x_1 = 0$$

Which gives the solution $x_1(t,T) = A(T)\cos\big(\ \omega_0 t + \phi(t)\big)$

Where $A(T), \phi(T)$

Slowing varying terms of x_1

Plugging it into the second equation we can solve for $x_0(t,T)$

Then plugging x_1,x_2

Into the third equation we would have an equation of the form

$$\partial_{tt}\ x_3 + \omega_0^2 = - - -$$

With the right hand side a some of some terms this then provide two ordinary differential equations for A, ϕ allowing one to solve for the equilibrium value of A as well as it's stability.

II. CONCLUSION

In this paper we have discussed Zero -Hopf bifurcation in a 3 D

Jerk systems and their behaviour in Jerk dynamical systems.

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